

Composite systems Bell correlations and prediction

PDT Foundations R06

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Purpose and dependencies

A last survivor law must work for jointly measured systems as well as isolated apparatus. This paper combines the quantum assumptions of R04 with an explicit choice for a conditional completion. It retains statistical independence of the hidden preparation variable from the freely selected measurement settings and permits the joint outcome law to be Bell nonlocal. This choice is labelled **B1**. It is a proposed model choice for review, not a theorem that every PDT completion must make that choice.

The additional composition premise **C1** identifies independent complex state spaces with their complex tensor product and local operations with the corresponding tensor factors. A4's single complex structure does not supply C1. Independent preparation, the available joint effects and the physical meaning of the tensor factors remain part of this bridge.

The Bell obstruction

Take binary outcomes $a, b \in \{-1, 1\}$, settings $x, y \in \{0, 1\}$ and a hidden antecedent λ . A Bell local model with measurement independence has

$$p(a, b | x, y) = \int p(a | x, \lambda) p(b | y, \lambda) d\mu(\lambda),$$

where the same μ is used for every setting pair. Define $E_{xy} = \sum_{a,b} ab p(a, b | x, y)$.

Proposition R06.1 The CHSH bound. Every such model obeys $|E_{00} + E_{01} + E_{10} - E_{11}| \leq 2$.

Proof. Conditional expectations A_x, B_y lie in $[-1, 1]$. For each antecedent the CHSH expression is $A_0(B_0 + B_1) + A_1(B_0 - B_1)$. Its absolute value is at most $|B_0 + B_1| + |B_0 - B_1| \leq 2$. Integrate with the same positive probability measure. ■

The theorem does not require determinism. Making A6 stochastic does not by itself evade it. A PDT model that retains this factorisation and measurement independence cannot reproduce Bell violating quantum correlations. The failure consequence belongs to that PDT model, not only to a discarded classical theory.

A precise conditional choice

Under C1, Q1 and the usual quantum preparation, let

$$p(a, b | x, y) = \text{Tr} \left[\rho \left(E_a^x \otimes F_b^y \right) \right].$$

For a singlet and unit measurement vectors x, y , this gives $p(a, b | x, y) = (1 - ab x \cdot y) / 4$. A uniform variable $u \in [0, 1]$, independent of x, y , can select a pair by partitioning the interval into these four probabilities in a fixed declared order. The selected pair is then a deterministic function of (x, y, u) .

This is an explicit mathematical representation of B1. Its joint antecedent law contains both settings and is not Bell local. It does not correlate the preparation variable with later setting choices. It also does not provide a local spacetime mechanism for gathering the settings, a covariant actualisation process, or an explanation of the quantum probabilities used to make the partition. Calling the pair a global relational event does not remove those missing dynamical questions.

For general finite settings one can use the same partition construction. R03's survivor keys or R05's supplied instrument may represent its record selection. None of these alternative samplers derives the measure or supplies relativistic dynamics. A future PDT completion could instead change measurement independence, but it must then replace B1 explicitly and derive the resulting testable joint law.

No signalling and the quantum bound

Proposition R06.2 Complete local statistics do not signal. A trace preserving operation on system B cannot change system A's outcome probabilities in the conditional tensor product model.

Proof. For a B channel E_B and an A effect E , the final probability is

$$\text{Tr}[(E \otimes E_B^*(I))\rho] = \text{Tr}[(E \otimes I)\rho],$$

because trace preservation is equivalent to $E_B^*(I) = I$. ■

This statement includes every record in the specified local instrument. Discarding no clicks, long delays or failed runs can produce setting dependent selected samples even when the complete data do not signal. A physical PDT claim must test the complete observable record, including any extra timing or trigger variables it makes accessible.

For dichotomic quantum observables with $A_x^2 = B_y^2 = I$, set

$$B = A_0 \otimes (B_0 + B_1) + A_1 \otimes (B_0 - B_1).$$

Direct expansion gives $B^2 = 4I - [A_0, A_1] \otimes [B_0, B_1]$. Each commutator has norm at most two, so $\|B\| \leq 2\sqrt{2}$. This is the Tsirelson bound. Pauli measurements on a maximally entangled state attain it. No signalling alone is weaker. Uniform binary outcomes constrained by $a \oplus b = x y$ give the CHSH value four while retaining uniform local marginals. Thus a no signalling postulate does not reconstruct quantum theory.

Why the complex structure must compose

Real coordinates can describe complex quantum mechanics if the complex structure is retained as additional structure. Simply taking ordinary real tensor products of independent real carriers is a different composition rule. For complex dimensions n and m , the real tensor product has dimension $4nm$, while the underlying real space of the complex tensor product has dimension $2nm$. The identification $(i v) \otimes w = v \otimes (i w)$ is substantive.

Renou and colleagues showed that real quantum theory with the specified independent source composition cannot reproduce certain complex quantum network correlations [B09]. Their 2021 paper proposes the separation, rather than reporting the later experimental test. The relevance to PDT is the composition obligation. A plane with $J^2 = -I$ at one site does not settle it. The result is

not a ban on expressing complex quantum theory in real notation with the needed global constraints retained.

Predicting a record without claiming omniscience

The selection model may be deterministic at a specified microscopic level while accessible records support only probabilistic forecasts. Conversely, an unresolved metaphysical account of actuality does not prohibit useful forecasts. Prediction needs its own declared information set and error criterion.

Let F_t be the information available before a forecast, let θ_0 be the fixed true model parameter and let Θ_t be an uncertainty set. Issue label a_t only if

$$\inf_{\theta \in \Theta_t} p_\theta(a_t | F_t) \geq 1 - \delta.$$

Assume these conditional laws are valid and $Pr(\theta_0 \notin \Theta_t) \leq \alpha$. If I denotes the event that a forecast is issued, then

$$Pr(\text{error} \cap I) \leq \delta Pr(I) + \alpha.$$

On the event that the parameter is covered, the conditional error is at most δ . On its complement the error probability is at most one. Splitting the expectation proves the bound. Consequently the conditional error among issued forecasts is bounded by $\delta + \alpha / Pr(I)$ when $Pr(I) > 0$. It is not generally bounded by $\delta + \alpha$. Adaptive stopping requires coverage valid at that stopping rule, such as an appropriate confidence sequence, rather than an unadjusted fixed time interval.

This is a statistical certificate under an identified model. It neither proves that every event is predictable nor proves a universal prohibition on prediction. An intervention that changes the apparatus can change the conditional outcome law. Merely knowing a prediction has not been shown here to force nature to change its answer.

Status and sources

The conditional quantum model satisfies the Tsirelson bound and complete record no signalling. The chosen hidden representation keeps measurement independence and rejects Bell locality. A6 alone makes neither choice. A covariant dynamical law for actual joint events remains open.

[B06] J. S. Bell, “On the Einstein Podolsky Rosen Paradox”, *Physics Physique Fizika* 1, 195–200 (1964). [DOI](#).

[B07] B. Hensen and colleagues, *Nature* 526, 682–686 (2015). [Author manuscript](#).

[B08] M. Giustina and colleagues, *Physical Review Letters* 115, 250401 (2015), [manuscript](#). L. K. Shalm and colleagues, *Physical Review Letters* 115, 250402 (2015), [manuscript](#).

[B09] M. O. Renou and colleagues, “Quantum Theory Based on Real Numbers Can Be Experimentally Falsified”, *Nature* 600, 625–629 (2021). [Author manuscript](#).

[S01] *PDT Foundations and Physical Closure*, 12 September 2026, Section 12.