

Gravity and conditional Einstein dynamics

PDT Foundations R10

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Purpose and dependencies

This paper states a viable conditional route from a common Lorentzian metric to Einstein dynamics. It also retains the September 23 counterexample showing why stable conserved link dynamics does not establish that route by itself. The internal metric of [R07](#) and the clock discussion of [R08](#) do not yet supply physical spacetime.

Use signature $(-, +, +, +)$ and $c = \hbar = 1$. The curvature convention is

$$R_{\sigma\mu\nu}^{\rho} = \partial_{\mu} \Gamma_{\nu\sigma}^{\rho} - \partial_{\nu} \Gamma_{\mu\sigma}^{\rho} + \Gamma_{\mu\lambda}^{\rho} \Gamma_{\nu\sigma}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{\mu\sigma}^{\lambda}.$$

with $R_{\sigma\nu} = R_{\sigma\rho\nu}^{\rho}$ and $G_{\mu\nu} = R_{\mu\nu} - R g_{\mu\nu} / 2$.

The necessary physical premises

G1 supplies a four dimensional smooth Lorentzian spacetime and a common metric interpretation for clocks, causal propagation and matter coupling. **G2** restricts the pure metric action to a local diffeomorphism invariant derivative expansion with at most two derivatives in total, modulo boundary terms, and a nonzero positive kinetic coefficient. **G3** specifies a matter action coupled to that metric and its total stress tensor. These are proposed conditional inputs. Their selection from the primitive relational dynamics remains open.

The optional premise **G4** supplies a Lorentz invariant scattering regime with a massless helicity two excitation, decoupling of unphysical polarisations, soft pole factorisation and momentum exchanging processes linking the matter sectors. It supports a conditional universality result. It is not needed merely to vary the G2 action.

A conserved link counterexample

Consider the already proposed positive hopping dynamics in its invariant zero matter mismatch sector. On a periodic cubic graph with spacing a , its link graph branch has dispersion

$$\hbar \omega_{\ell}(k) = 4 \kappa \sum_{r=1}^d \sin^2(k_r a / 2) = \kappa a^2 |k|^2 + O(a^4 |k|^4).$$

One way to verify this is to use the unsigned matter vertex to edge incidence matrix C . For a graph of degree $2d$, $C C^T = 2d I + A$ and the link Laplacian is $K = \kappa (4d I - C^T C)$. A matter Fourier vector has adjacency eigenvalue $2 \sum_r \cos(k_r a)$. Applying K to its nonzero image under C^T gives the displayed branch.

The uniform mode has zero energy and the positive number conserving Hamiltonian is stable. Nevertheless the excitation has $\omega \sim |k|^2$, not the $\omega \sim |k|$ of a relativistic massless field. Nonsingular constant calibrations of time and distance cannot change that exponent. A different collective sector might behave differently, but its existence and selection need a proof.

This counterexample defeats the implication from conserved stable links alone to universal relativistic gravity. It does not refute every possible PDT gravitational completion.

Conditional universality of leading coupling

Under G4, the leading soft spin two emission factor has the form

$$M_{n+1} \approx \sum_i \eta_i \gamma_i \frac{p_i^\mu p_i^\nu \epsilon_{\mu\nu}}{p_i \cdot q} M_n.$$

The signs η_i distinguish incoming and outgoing legs. Decoupling a polarisation shift $\epsilon_{\mu\nu} \mapsto \epsilon_{\mu\nu} + q_\mu \xi_\nu + q_\nu \xi_\mu$ requires $\sum_i \eta_i \gamma_i p_i^\nu = 0$. In elastic scattering between species A and B, momentum conservation reduces this to

$$(\gamma_A - \gamma_B)(p_A' - p_A)^\nu = 0.$$

Nonzero momentum transfer therefore makes the couplings equal. Connected scattering channels propagate this equality through the interacting sector. This is the established leading soft graviton argument [B11]. It does not derive the Lorentz regime or the helicity two field assumed in G4. It also leaves finite wavelength nonminimal couplings and higher derivative corrections to be analysed.

Conditional action classification

Theorem R10.1 Two derivative pure metric action. Within G1 and G2, the bulk gravitational action is

$$S_g = \frac{M^2}{2} \int d^4x \sqrt{-g} (R - 2\Lambda), M^2 > 0,$$

up to boundary terms, with undetermined constants M and Λ .

Proof. In normal coordinates first derivatives of the metric vanish at a point. At zero derivative order the metric alone supplies only a constant scalar. Tensorial second derivative data are curvature. At total derivative order two, a nonzero scalar contraction linear in curvature reduces to R . Terms with two or more curvature factors have at least four derivatives in total and are excluded by G2. Total derivatives change only the boundary terms. Thus the integrand is $\sqrt{-g}(c_0 + c_1 R)$, with $M^2 = 2c_1$ and $\Lambda = -c_0/(2c_1)$. ■

This proof uses a narrower action class than merely saying the equations involve second derivatives. Lovelock's theorem is the related classification of divergence free metric tensors under its own locality, dimensional and differential assumptions [B12]. It cannot be used to derive those assumptions from A0 through A6. An effective leading action and an exact action with no higher curvature terms are also different claims.

Define $T_{\mu\nu} = -2(\sqrt{-g})^{-1} \delta S_m / \delta g^{\mu\nu}$. With an appropriate boundary treatment, variation gives

$$M^2(G_{\mu\nu} + \Lambda g_{\mu\nu}) = T_{\mu\nu}, M^2 = (8\pi G)^{-1}.$$

The contracted Bianchi identity requires $\nabla^\mu T_{\mu\nu} = 0$. For a minimally coupled point particle, varying $-m \int d\tau$ gives the geodesic equation. The mass factor cancels, so this ideal test particle motion is universal within the stated coupling. Finite size, spin and nonminimal couplings require their own analysis.

Record events and stress conservation

An actual record update cannot be coupled to gravity merely by replacing a state on the right hand side of Einstein's equation. The total stress must remain conserved, including apparatus, reservoir and any record mechanism. A jump in a selected matter expectation without compensating stress can violate the Bianchi constraint.

Even in finite quantum mechanics, ensemble energy conservation for every state under an unread instrument requires

$$\sum_{a,\alpha} L_{a\alpha}^\dagger H L_{a\alpha} = H.$$

The equivalence follows by taking the trace against every density matrix. It does not hold for every measurement. With $H = \sigma_x$, a projective measurement in the σ_z basis sends an initial $|+\rangle$ mean energy from one to zero in these arbitrary units. Apparatus work must account for the difference in a physical model. Ensemble equality, even when it holds, does not by itself establish energy conservation in each realised trajectory.

Thus R05 and the present Einstein completion are compatible targets for a joint model. They are not yet a constructed coupled quantum gravity theory. The missing common stress and record dynamics is a physical obligation before the question of ultimate actuality.

Numerical observational anchors

The following are specific published benchmarks, not a claim to list the newest bounds in every model. A proposed deviation must be translated into the tested observable with its domain and confidence convention stated.

Observable	Published result	Interpretation required
Cassini time delay	$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$	PPN parameter in the 2003 radio link analysis
GW170817 speed comparison	$-3 \times 10^{-15} \leq (c_g - c) / c \leq 7 \times 10^{-16}$	2017 bound under the paper's emission timing assumptions
MICROSCOPE free fall	$\eta_{Ti, Pt} = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$	2022 titanium and platinum comparison. Statistical uncertainty at one sigma, then systematic uncertainty.

These values constrain a model's predictions. They are not values derived by PDT. A decisive disagreement falsifies the relevant completion. It reaches the primitive foundations only if the failed consequence is unavoidable under their indispensable commitments.

Sources

[S08] *PDT Universal Geometry and Einstein Action Proof Attempt*, 23 September 2026. The link dispersion, conditional soft coupling and action classification are retained with their stated limits.

[B11] S. Weinberg, “Photons and Gravitons in S Matrix Theory”, *Physical Review* 135, B1049–B1056 (1964). [DOI](#).

[B12] D. Lovelock, “The Einstein Tensor and Its Generalizations”, *Journal of Mathematical Physics* 12, 498–501 (1971). [DOI](#).

[B13] B. Bertotti, L. Iess and P. Tortora, *Nature* 425, 374–376 (2003). [Original paper record](#).

[B14] LIGO, Virgo, Fermi GBM and INTEGRAL collaborations, “Gravitational Waves and Gamma rays from a Binary Neutron Star Merger”, 2017. [arXiv 1710.05834](#).

[B15] P. Touboul and colleagues, “MICROSCOPE Mission Final Results”, *Physical Review Letters* 129, 121102 (2022). [arXiv 2209.15487](#).