

Selection and persistent records

PDT Foundations R03

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Purpose and dependencies

This paper uses A5 and A6 from R01. It gives exact conditions for a last survivor, proves what finite selection can achieve, and supplies a simple physical stability model. These results explain different aspects of a record. The claim that a unique admissible continuation is actual remains the adopted A6 boundary.

The first singleton

Fix a finite candidate set C of size $m \geq 2$. Give candidate a an antecedently determined loss time $\ell_a \in (0, \infty)$ and define

$$A(s) = \{a : s < \ell_a\}, s \geq 0.$$

Theorem R03.1 Last survivor criterion. A singleton occurs before extinction if and only if the largest loss time is unique. If $\ell_{(m)}$ is that largest value and $\ell_{(m-1)}$ the second largest, the first singleton is attained at $s_* = \ell_{(m-1)}$. It lasts on $[s_*, \ell_{(m)})$ before the process is stopped.

Proof. At s_* every other candidate has a loss time at most s_* and is excluded, while the largest survives. At every earlier stage the candidate with second largest loss time and the largest both survive. If at least two candidates tie for the greatest loss time, they survive together until all remaining candidates vanish together. There is then no singleton. ■

Several smaller loss times may tie without affecting the conclusion. The one candidate case resolves at the initial stage. Distinct continuously distributed random keys make a largest tie a probability zero event, but they do not establish resolution for every possible input. An all input theorem must handle ties separately or restrict its domain.

For discrete development, use integer loss indices and an attained minimum. For continuous development, the theorem proves attainment for this particular process. It does not license replacing an arbitrary limiting singleton by an event. For example $A(s) = [a, b)$ for every $s < 1$, followed by $A(1) = \emptyset$, has no realisation under A6.

Symmetry and finite selection

Proposition R03.2 The finite orbit criterion. A group action on an admissible set admits a nonempty finite invariant subset exactly when it has a finite orbit.

Proof. A finite orbit is such a subset. Conversely, the orbit of any member of an invariant finite subset lies inside that subset and is finite. ■

If an invariant rank takes values in the nonnegative integers and has a nonempty finite minimum fibre, that fibre gives FCS. This does not follow from the existence of a rank alone. Its minimum fibre may be infinite.

Theorem R03.3 Equivariant single selection. Let a symmetry group act on data and their admissible candidate sets compatibly. On the orbit of a datum x , an equivariant admissible selector exists exactly when the stabiliser of x fixes a candidate admissible at x .

Proof. If f is a selector and $gx = x$, then $gf(x) = f(gx) = f(x)$. Thus $f(x)$ is stabiliser fixed. Conversely, choose a candidate a fixed by the stabiliser and define $f(gx) = ga$. If $gx = hx$, then $h^{-1}g$ stabilises x and fixes a , so the definition is independent of the representative. It is equivariant by construction. ■

Two perfectly interchangeable candidates therefore cannot be uniquely selected by a deterministic rule that preserves all the symmetries of the supplied data. Candidate attached marks can break the symmetry, but their origin and preparation then become part of the model. This result does not show that nature cannot realise a record. It shows what an equivariant mathematical selector needs as input.

A probability model and its freedom

Let $w_a > 0$ with $\sum_a w_a = 1$. Adopt independent uniform variables U_a and keys $K_a = U_a^{1/w_a}$. Use these keys as loss times and stop at the first singleton.

Proposition R03.4 Weighted survivor law. The unique largest key has label a with probability w_a .

Proof. Its distribution function is $F_a(k) = k^{w_a}$ on $[0, 1]$. Integrating against all competing distribution functions gives

$$Pr(a \text{ wins}) = \int_0^1 w_a k^{w_a-1} \prod_{b \neq a} k^{w_b} dk = w_a.$$

The independent continuous laws make ties null. Zero weight candidates may be removed before the construction. ■

If a coarse capacity is split into positive parts, taking the maximum of independent refined keys preserves its distribution because $k^u k^v = k^{u+v}$. More generally, under independent maximum aggregation and continuous weight dependence, $F_{u+v} = F_u F_v$ implies $F_w = F_1^w$ wherever $0 < F_1 < 1$. Take a negative logarithm, use additivity and monotonicity in the weight, and approximate real weights by rationals. A common strictly increasing change of key scale leaves the winner unchanged. The baseline scale is not selected.

This is a standard weighted sampling construction, retained from the earlier PDT analysis. It yields Born probabilities only if the weights are independently identified as Born weights. A6 alone permits every probability vector. Supplying a sampler after choosing the answer distribution is a consistency construction, not a derivation of that distribution from phase physics.

There is also a useful exact error estimate. Let $Y_a = -\log K_a$, so the Y_a are independent exponentials of rates w_a . If G is the gap between their smallest and second smallest values, then

$$Pr(G > t) = \sum_a w_a e^{-(1-w_a)t}.$$

To prove this, integrate the density for $Y_a = y$ against the probability that every other value exceeds $y + t$. The integral equals $w_a e^{-(1-w_a)t}$ and the disjoint winning events sum. If every measured score has error at most η , the winning label cannot change whenever $G > 2\eta$. Therefore

$$Pr(\text{wrong winner}) \leq 1 - \sum_a w_a e^{-2\eta(1-w_a)}.$$

The estimate assumes the adopted key distribution and a certified uniform score error. It is not a universal PDT error law.

A record can be stable without resolving actuality

A5 is historical bookkeeping. Physical persistence requires a device and dynamics. Consider a dimensionless pointer coordinate under a supplied dissipative law,

$$\frac{dq}{ds} = q - q^3 + u(s), |u(s)| \leq \varepsilon < 3/8.$$

Assume a measurable bounded forcing and the ordinary Carathéodory solution. Interpret the positive and negative regions as two record values.

Proposition R03.5 Conditional record stability. Each interval $[1/2, 3/2]$ and $[-3/2, -1/2]$ is forward invariant. A pointer prepared in either interval retains its sign for as long as the forcing bound and law hold.

Proof. At $q = 1/2$, the unforced derivative is $3/8$, so even the most negative permitted forcing leaves it positive. At $q = 3/2$ it is $-15/8$, so it remains negative. At the two negative boundaries the signs reverse. The vector field points into each interval at its boundary. Uniqueness and the first exit argument establish invariance, also for bounded measurable forcing by the standard integral solution formulation. Boundedness within the interval prevents finite time blowup. ■

The write operation that places the pointer in a chosen interval has not been derived. Nor does this dissipative equation contain its own thermal environment or energy balance. These are model inputs. The proposition shows that one can ask a precise question about persistence after a record exists without answering why that record became actual.

A complete physical application must also say what happens when the device fails. A burned notebook does not undo its past. A5 retains the historical event, while the readable present memory may be lost. Confusing these two notions would make the axiom empirically untenable.

Boundary and next paper

PDT can adopt the statement that the last physically admissible continuation is realised. The present mathematics establishes when a specified process has such a continuation and when a specified memory model retains a sign. It does not establish that every physical episode reaches a singleton, or that all records persist forever.

R04 now introduces the physical quantum bridge needed to constrain probabilities and operations. That bridge is additional to the selection principles proved here.

Sources

[S01] *PDT Foundations and Physical Closure*, 12 September 2026, Sections 2, 5 and 6.

[S03] *PDT Selection Identifiability and Physical Closure*, 11 September 2026, Sections 1 and 2.

The weighted maximum construction and its baseline freedom are retained with their original restricted scope. The pointer interval example is an explicit stability witness in this reconstruction, not a claimed microscopic PDT law.