

## Joint closure and the realisation boundary

### PDT Foundations R12

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### Purpose and finding

The preceding papers establish a consistent structural core and several nonempty conditional physical models. They do not establish one closed microscopic theory that selects quantum measurement, matter, gravity and cosmology together. This paper gives the exact scope of the achieved closure and a constructive test for the missing joint model.

Graham's central philosophical position remains visible. A realised world has records, and the last physically admissible continuation is taken to become one of them. A6 states that commitment. The ultimate reason why an actual event exists, rather than remaining a mathematical possibility, is left outside the claimed derivation. This boundary must not be used to excuse missing equations for measurable record formation, stable memory, energy exchange or propagation.

### What has closed mathematically

R01 gives explicit structures satisfying the formal axioms. R02 permits loop phase and noncommuting full transport, proves the minimal phase plane and identifies limits of the regular representation. R03 proves finite selection criteria and conditional memory stability. R04 supplies finite quantum probability and reversible evolution once its operational bridge is adopted. R05 supplies exact finite exclusion histories and their limitations. R07 supplies a conditional oriented metric carrier. R10 classifies a conditional Einstein action, and R11 supplies explicit cosmological evolution.

These results have their own domains and premises. The word *conditional* does not make them worthless. It identifies which part of the explanation has been provided and which physical choice remains to be justified.

**Proposition R12.1 Consistent finite quantum histories.** Given an initial density matrix and a finite instrument at each finite history, the trace of the composed unnormalised operations defines normalised consistent probabilities on the record tree.

**Proof.** Let  $\rho_h$  be the unnormalised state at a prefix  $h$ . Completeness gives  $\sum_a \text{Tr } I_a^h(\rho_h) = \text{Tr } \rho_h$ .

Starting from trace one, induction normalises every finite level and gives the correct prefix marginal. Zero probability prefixes can receive arbitrary later conditional definitions without changing the measure. With finitely many outcomes per node, these consistent cylinder probabilities define a probability law on infinite paths by the usual countable extension construction. ■

R05 represents each supplied instrument by a finite last survivor sequence. A conditional history law is therefore mathematically available. This says nothing about the physical clock accumulation of infinitely many records. A lower bound on episode separation would exclude finite time accumulation, but that bound is another physical premise. The theorem also leaves the instrument, its environment, its energy budget and the interpretation of one path as actual supplied.

## Separate witnesses do not make a joint witness

Let  $q$  denote one common physical configuration after gauge and unit redundancies have been removed. Suppose candidate requirements are explicit residuals

$$C(q) = (C_{state}, C_{record}, C_{matter}, C_{geometry}, C_{conservation}, C_{limit}).$$

Every component must refer to the same variables and observation map. Separate solutions of separate equations do not imply a common solution. On  $[0, 1]$  the sets  $\{0\}$  and  $\{1\}$  are individually nonempty but disjoint. With strictly positive weights  $\lambda_i$ , the total burden  $\sum_i \lambda_i \|C_i(q)\|^2$  has zero exactly at a common zero, but its minimum can be positive. Defining such a burden does not produce a zero.

Compactness and the finite intersection property can prove a common point for a family of closed sets. For a finite list, however, the finite intersection hypothesis already includes the full list. It cannot replace the missing compatibility calculation.

**Proposition R12.2 A constructive local certificate.** Let  $C: \mathbb{R}^n \rightarrow \mathbb{R}^n$  be continuously differentiable on a neighbourhood of a closed ball  $B(q_0, R)$ . Use a vector norm and its induced operator norm. Suppose an invertible matrix  $A$  and a number  $\kappa < 1$  satisfy

$$\sup_{q \in B} \|I - ADC(q)\| \leq \kappa, \quad \|AC(q_0)\| \leq (1 - \kappa)R.$$

Then  $C$  has a unique zero in the ball. For every  $q$  in it,

$$\|q - q_*\| \leq \frac{\|AC(q)\|}{1 - \kappa}.$$

**Proof.** The map  $T(q) = q - AC(q)$  is a contraction of factor  $\kappa$ . The centre bound makes it map the ball into itself. Iteration gives a unique fixed point. Invertibility of  $A$  makes that point a zero of  $C$ . The triangle inequality  $\|q - q_*\| \leq \|q - Tq\| + \kappa \|q - q_*\|$  gives the estimate. ■

For an overdetermined system, a square subsystem certificate must be followed by verification of every remaining residual at the certified zero. A numerical root without controlled derivative bounds is not this certificate. No full physical residual map satisfying these tests has been supplied by the current sources or constructed in this suite.

## Existence and identifiability differ

At a common zero, full column rank of  $DC$  on the physical quotient gives local isolation and a local residual bound. The smallest singular value is positive, so differentiability bounds  $\|C(q)\|$  below by a positive multiple of  $\|q - q_*\|$  nearby. It does not imply global uniqueness. The function  $x^2 - 1$  has two regular isolated roots. Conversely  $x^3$  has an isolated root with zero derivative, so a rank failure does not by itself prove a family of solutions.

Appending a residual  $F(C(q))$  with differentiable  $F$  and  $F(0) = 0$  changes neither the common zero set nor derivative rank there. Its derivative rows are combinations of the old rows. A new name for an old constraint supplies no new selection. This is particularly relevant to proposals that one final closure equation must determine all constants without writing its independent content.

## Interfaces that a common model must satisfy

| Interface                         | Required common object                                     | Present position                                      |
|-----------------------------------|------------------------------------------------------------|-------------------------------------------------------|
| Relational data to quantum states | A physical map and allowed operations                      | Q1 through Q4 supplied conditionally                  |
| Quantum amplitudes to records     | An antecedent trigger, environment and complete record law | Ideal instruments constructed, intrinsic trigger open |
| Records to physical memory        | Write process, stability and failure law                   | Conditional stability witness only                    |
| Internal metric to spacetime      | Displacement, clock and causal identification              | S2, T1, T2 and G1 supplied conditionally              |
| Matter and gravity                | One action and conserved total stress                      | No complete record coupled witness                    |
| Finite model to continuum         | Uniform control and identified observables                 | Specific obstruction proved, general limit open       |
| Calibration to prediction         | Independently selected dimensionless relation              | Conversion closed, constants unselected               |

The table identifies equations and constructions still needed. It is not another list of postulates being declared true to obtain a master closure theorem.

## The boundary Graham has identified

There are two meanings of survival. Mathematical persistence means that a history keeps its prefix or that a pointer remains within a stable region. Physical actuality means that one event has occurred. A6 connects a sole admissible continuation with the second meaning. Neither a stability theorem nor a Born probability proves that connection.

Graham's use of *God* expresses the ultimate explanatory boundary. The papers do not introduce God as a defined mathematical variable, a measurable cause or a device for choosing outcomes. They also do not require a definition of God before completing ordinary physical questions about clocks, stress or apparatus. Those questions remain within the theory's obligations.

The philosophical observation that intervention can change an outcome is compatible with ordinary context dependent dynamics. A stronger claim that a known prediction necessarily changes its own outcome would require a precise law and experiment. None is inferred from the metaphor. The familiar historical remarks about dice are not treated as statements of general relativity or quantum mechanics.

PDT may legitimately stop at an adopted realisation principle while remaining a useful conditional theory. It cannot claim that this is its only remaining boundary until the independent dynamical and identification gaps have been closed.

## Conditions for abandonment and replacement

An inconsistency among indispensable formal axioms requires changing that foundation. A counterexample to a theorem requires withdrawing the implication. A failed experiment tests a specified physical completion with its declared preparations, parameters and uncertainties. It reaches the entire programme only when the failed prediction follows unavoidably from the indispensable commitments.

There are also explanatory failures. Recovering a target law solely by assuming an equivalent law does not derive it from a simpler basis. Fitting every target independently does not establish universal selection. If the research claim is that PDT independently determines a constant or mechanism, these failures defeat that claim even when a compatible conditional model remains useful.

A next discriminating test should freeze one complete record model, independently calibrate its permitted parameters, specify every retained and discarded trial, and predict held out observations. If it agrees identically with quantum mechanics for every allowed intervention, report it as an interpretation or representation with an equivalence proof. If an intrinsic trigger changes an echo or timing law, that difference must follow from its specified dynamics rather than from an after the fact choice of collapse time.

## Conclusion for review

The suite closes the formal meaning of the axioms and a set of explicit conditional constructions. It does not close the full physical theory. The strongest next step is a common microscopic record model with its state map, apparatus coupling, reservoir, conserved stress and robust admissibility rule. It should be solved and checked before its mechanism is extended to matter, gravity or cosmology. [R13](#) provides the audit record and reproducible checks for the mathematics already stated.

## Sources

[S01] *PDT Foundations and Physical Closure*, 12 September 2026, Sections 14 through 17. Common witness and identifiability results, and withdrawal of the earlier assembly only closure theorem.

[S07] *PDT Physical Closure Gap and Refinement Test*, 18 September 2026. Independent physical freedoms and the need for a specified refinement trajectory.

[S08] *PDT Universal Geometry and Einstein Action Proof Attempt*, 23 September 2026. Conditional gravitational results and their prerequisites.