

Core axioms and consistency

PDT Foundations R01

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Review draft 1.0 • 24 September 2026

Purpose and result

PDT begins with relations and asks whether the familiar objects of physics can arise from their organisation. This paper gives that starting point a precise mathematical form. It preserves the distinction already present in the September 12 foundations between full relational transport and its phase information. It also keeps the last coherent survivor principle as a physical commitment. A mathematical description of a survivor does not, by itself, explain actuality.

The package below has explicit mathematical models. It permits nontrivial loop phase and noncommuting transport. It does not select a Hamiltonian, a probability measure, physical dimension or gravitational law. These limitations are proved by constructions in this and the following papers. They are not repaired by treating the desired conclusions as unnamed assumptions.

Throughout the suite, a **structural result** follows from the stated mathematical data. A **conditional physical result** also uses a declared identification with preparations, records, clocks or fields. An **adopted principle** is an input. An **open obligation** has no completed derivation here. Mathematical consistency is always relative to the ordinary set theory and analysis used in the constructions.

The primitive package

A0 Distinction. There is a set X with at least two elements. No distance, duration, particle identity or spatial embedding is included in this assertion.

$$|X| \geq 2.$$

A1 Phase. The phase group is $T = U(1) = \{z \in \mathbb{C} : |z| = 1\}$, with its usual multiplication and topology. This is an adopted choice of relational variable. It is stronger than bare distinction and does not follow from A0. Complex notation here describes the circle group, which can equally be defined by rotations. It does not assume physical complex probability amplitudes.

A2 Coherent comparison. There is a small groupoid G with object set X and a functor $\phi : G \rightarrow BT$. Here BT is the one object groupoid whose arrows are phases. For $g : x \rightarrow y$ and $h : y \rightarrow z$,

$$\phi(h \circ g) = \phi(h) \phi(g), \phi(1_x) = 1, \phi(g^{-1}) = \phi(g)^{-1}.$$

Composition is associative on composable arrows. Each object has an identity and each arrow has an inverse. These are comparisons. An inverse comparison is not a physical operation that deletes a realised record.

The functor is not required to distinguish all arrows. Different relational transports may carry the same scalar phase. In particular, no surjectivity or faithfulness condition on the whole functor is hidden in A2. Applications requiring a sufficiently rich realised phase image must state that condition.

A3 Faithful encoding of phase. Phase may be encoded by its angle class in $\mathbb{R}/(2\pi\mathbb{Z})$. The encoding distinguishes phase elements and respects addition modulo a full turn,

$$c(zw) = c(z) + c(w) \pmod{2\pi}.$$

This faithfulness concerns the group T . It does not say that $c \circ \phi$ is injective on the arrows of G . All identity arrows legitimately receive angle zero. A3 is retained as a legacy label for an algebraic convention already available from A1. It is not an independent law selecting physical dynamics. A real angle without the quotient requires a specified local lift or additional winding information.

A4 Minimal real phase carrier. Represent T faithfully by continuous real linear transformations on a nonzero finite dimensional real vector space. Within this explicitly chosen class, use the smallest possible dimension. R02 proves that the dimension is two and that the carrier has a complex structure. Linearity, continuity and the minimisation criterion are inputs. No spatial dimension follows from this carrier dimension.

A5 Retained history. A realised history is an ordered finite sequence $H_n = (r_1, \dots, r_n)$ with a specified admissible extension set. Appending a record preserves the existing prefix,

$$H_{n+1} = H_n \parallel r_{n+1}, \text{prefix}_n(H_{n+1}) = H_n.$$

For every allowed appended record the append map is injective on its domain. This describes retained historical identity. It does not prove that a physical memory device is permanently indestructible, that arbitrary relational presentations embed into later ones, or that every history admits a next event. Physical memory stability is treated separately in R03.

A6 Last coherent survivor realisation. For an episode following H_n , a physical model must specify a finite exhaustive set C_n of mutually exclusive candidate records, the relevant antecedents, environment and governing law. It must specify the admissibility process from those inputs rather than from the eventual observed answer. Let s be its development parameter and let $A_n(s) \subseteq C_n$ decrease by inclusion as the episode develops. If there is a first attained finite stage s_* at which exactly one candidate remains, with no earlier extinction, the sole candidate is realised and appended to the history. The nesting condition below applies whenever $s_1 \leq s_2 \leq s_*$ within the episode.

$$A_n(s_2) \subseteq A_n(s_1), A_n(s) \neq \emptyset (s \leq s_*), A_n(s_*) = (r_{n+1}).$$

After the event, the episode stops and the record is retained under A5. The next episode receives its own antecedents. Extending an illustrative loss rule beyond the event until its final candidate also disappears is not part of this stopped rule.

A6 is an adopted physical realisation principle. It neither constructs the admissibility law nor ensures that an arbitrary law terminates. It has no probability distribution, universal rate or automatically chosen detector basis. It is deterministic only if the fully specified antecedents and law determine the admissibility history. The phrase *last coherent survivor snaps* refers here to the A6 event. Coherence must be given an operational or dynamical definition in each physical application.

Time labels and scope

The integer n counts completed records. The variable s describes progress inside one episode. Its domain may be discrete or continuous and must be declared by the model. In a discrete domain with a first singleton the event index is an attained minimum. In a continuous domain an infimum need not be attained. Thus writing an infimum is insufficient to prove an event.

Neither label is yet elapsed laboratory time or relativistic proper time. A clock map belongs to R08. This separation removes the earlier ambiguity between record number and a continuous parameter without assuming a physical continuum at the bottom of the theory.

Models and limits

Proposition R01.1 A noncommuting model with phase. The structural package A0 through A5 has a model for every finite number $N \geq 2$ of objects, with full transport containing both continuous phase and noncommuting operations.

Proof. Let $X = \{1, \dots, N\}$ and $K = T \times S_3$. An arrow is (x, k, y) from x to y . Define

$$(y, k_2, z) \circ (x, k_1, y) = (x, k_2 k_1, z).$$

Use (x, e, x) as identity and (y, k^{-1}, x) as inverse. The group laws of K establish every groupoid identity. Let $\phi(x, (z, \sigma), y) = z$. Projection to the first factor is a functor. Rotations of the real plane give the A4 representation. Finite words with ordinary append give A5. The permutations (12) and (23) do not commute, although both have phase one. This is a consistency witness, not a derivation of the observed gauge groups. ■

Proposition R01.2 A terminating extension exists. The preceding model can be equipped with episodes satisfying the antecedent conditions of A6 at every record number.

Proof. For an episode with at least two candidates, give every candidate a distinct, antecedently supplied positive key. During the episode retain candidate a exactly while $s < k_a$. At the second largest key only the largest label remains. Stop there and append it. A one candidate episode resolves immediately. Repeating this construction produces arbitrarily long histories. R03 states the more general theorem and its tie exceptions. ■

The construction establishes that the formal constraints do not contradict one another. Assigning the word *realised* to an appended mathematical label is not a proof that the physical world obeys A6. That physical commitment remains explicit.

Proposition R01.3 Why full additive faithfulness would be too strong. If a group admits an injective homomorphism into an abelian additive group, it is abelian.

Proof. For any g, h , the images of gh and hg are equal because their coordinate sums commute. Injectivity implies $gh = hg$. ■

Applied to an isotropy group $G(x, x)$, this shows why the old phrase *faithful additive coordinate for every relation* cannot be retained if noncommuting transport is intended. Under A1 through A3 only the phase quotient is abelian. Matrix interactions will need additional representations and dynamics, as R09 makes explicit.

Independent finite selection

Finite Canonical Selection, abbreviated FCS, is not included in A0 through A6. When used, it asserts the existence of a nonempty finite admissible set invariant under a declared symmetry action. It selects a set, not necessarily one member. R03 proves the exact finite orbit criterion and the obstruction to selecting a single element from completely symmetric data.

No axiom here says that every desired physical law has a compatible realisation. The suite begins with explicit nonempty structures, develops their consequences, and adds physical assumptions only at named interfaces. R02 treats phase transport. R03 treats selection and histories. Neither paper assumes the quantum or gravitational conclusions that appear later.

Sources

[S01] *PDT Foundations and Physical Closure*, second review edition, 12 September 2026, Sections 2 and 3. Internal source file PDT_Foundations_and_Physical_Closure_2026-09-12.docx. The arrow phase and restricted meaning of faithfulness restore this source rather than introducing a new claim of priority.

[B01] Tom Leinster, *Basic Category Theory*, 2014. Author manuscript [arXiv 1612.09375](https://arxiv.org/abs/1612.09375). Groupoids, functors and the usual hom set meaning of faithfulness.