

Interactions matter and refinement

PDT Foundations R09

Prepared for Graham Fincham and Daniel Hilton

Review draft 1.0 • 24 September 2026

Purpose and dependencies

The corrected phase axioms permit richer transport than one scalar phase. They do not determine its internal degrees of freedom or interactions. This paper gives concrete compatible constructions, identifies what they assume and carries a useful refinement obstruction into the rebuilt suite. It uses R02's transport distinction, R04's conditional state interpretation and R08's calibration.

The premise **M1** specifies matter multiplets, their representations and a microscopic action. **M2** specifies a regulator family and the physical observables to be held fixed as that regulator is removed. Existence of a limit, its spectrum and its parameters are conclusions to prove from these data, not additional assumptions counted as completed physics.

Noncommuting transport is permitted but not derived

A2 allows full isotropy groups with noncommuting elements because the phase functor need not be faithful on all arrows. In a two component complex carrier the matrices σ_x and σ_z generate noncommuting rotations. Their commutator supplies the remaining Pauli direction. Thus a phase rotation together with an independently supplied mixing operation can generate an $SU(2)$ action.

The mixing operation and the two component physical carrier are extra data. Repeating the word phase does not produce them. Similarly an $SU(3)$ colour multiplet can be represented consistently, but its dimension, representation content, chiral fermions and couplings have not been selected by A0 through A6. A3's corrected scope keeps this route open. It does not complete it.

A finite gauge action witness

Choose a finite lattice, a positive integer N , link variables $U_{xy} \in SU(N)$ with $U_{yx} = U_{xy}^{-1}$, and vertex transformations $V_x \in SU(N)$. Set $U_{xy} \mapsto V_y U_{xy} V_x^{-1}$, using path products in traversal order. A plaquette product changes by conjugation at its base point. Consequently

$$S_g = \beta \sum_p \left(1 - \frac{1}{N} \operatorname{Re} \operatorname{Tr} U_p \right), \beta \geq 0,$$

is gauge invariant and nonnegative. Each eigenvalue of U_p has unit modulus, so its real trace is at most N . On a finite lattice the product Haar measure is finite, the action is continuous and bounded, and $\int e^{-S_g} dU$ is strictly positive. This supplies a well defined finite Euclidean gauge measure.

It is a standard Wilson type construction, not a derivation of physical QCD. The lattice, N , β , matter species and observation map are M1 inputs. A finite positive measure alone supplies neither a continuum Lorentzian theory nor reflection positivity for an arbitrary added matter action.

A vacuum is not yet matter or a mass gap

Calling a stable configuration a vacuum identifies a state relative to chosen dynamics. Its excitations need an energy, momentum and observable interpretation before they can be identified as particles. A classical strict local minimum with positive Hessian supplies local stability under suitable dynamics. It does not select the observed particle spectrum.

For an elementary counterexample, let a bounded positive self-adjoint operator on ℓ^2 have eigenvalues $0, 1, 1/2, 1/3, \dots$. Its normalised zero eigenvector is unique up to phase, yet positive energies approach zero. Hence a unique ground state does not imply a positive spectral gap. Finite truncations have positive gaps that shrink to zero, so finite numerical gaps also need a uniform lower bound before a continuum gap is claimed.

Stable sector actions do not fix calibrated mass ratios either. Let $S_e(x) = I_e + x^2$ and $S_\mu(y) = I_\mu + y^2$, with positive I_e, I_μ . Both minima have the same positive Hessian. Varying I_μ leaves the electron calibration and both local stabilities unchanged while changing I_μ/I_e . Extra constraints may remove this freedom, but they must actually be supplied.

These countermodels concern the proposed implications. They do not assert that the physical electron or muon follows these illustrative potentials.

A gauge uniform refinement obstruction

The September 18 electromagnetic analysis specifies a finite four dimensional Euclidean Wilson matrix

$$M_t(U) = I - t H(U), 0 < t < 1/8.$$

The hopping operator is

$$H_{xy} = \sum_{\mu=1}^4 \left[(I - \gamma_\mu) U_{x\mu} \delta_{y, x+\hat{\mu}} + (I + \gamma_\mu) U_{x-\hat{\mu}, \mu}^* \delta_{y, x-\hat{\mu}} \right].$$

The Euclidean gamma matrices are Hermitian and obey the Clifford relations. Links have unit modulus, boundaries are open and missing hops are omitted. This four dimensional regulator is supplied by M1. It is not R07's derived physical spacetime.

Theorem R09.1 Uniform decay on the declared branch. Let $d(x, y)$ be graph distance and $\rho = 8t < 1$. Then for every link configuration

$$\| [M_t(U)^{-1}]_{xy} \| \leq \frac{\rho^{d(x,y)}}{1-\rho}.$$

The same bound holds after any normalised positive gauge average with a dressing of modulus at most one.

Proof. Write $P_\mu^\pm = (I \pm \gamma_\mu)/2$. One directional contribution is $2(P_\mu^- T_\mu + P_\mu^+ T_\mu^\dagger)$, where the covariant partial shift has norm at most one and commutes with the spin projectors. The orthogonal spin blocks give directional norm at most two, hence $\| H \| \leq 8$. The Neumann series $\sum_{n \geq 0} t^n H^n$ gives the norm convergent inverse. A term cannot join x to y in fewer than $d(x, y)$ hops. Bound each

remaining block by 8^n and sum. Positivity of the measure and the triangle inequality give the averaged bound. ■

For the specific measure proportional to $\det M_t(U) dU$, gamma five Hermiticity makes the determinant real. The norm bound prevents a zero for $t < 1/8$. Continuity from determinant one at $t=0$ therefore makes it positive. A finite real gauge action can also be included while preserving positivity of this finite measure. This determinant statement alone is not a transfer matrix theorem.

At lattice spacing a , the corresponding inverse correlation length in this specified channel is bounded below by $-\log(8t)/a$. At every fixed $t < 1/8$ that lower bound diverges as $a \rightarrow 0$. The channel cannot retain a finite physical correlation length along such a fixed hopping trajectory. No inference about every possible composite or every PDT completion follows from this one channel bound.

For the declared free zero field convention, $E_{lat} = \log(1/(2t) - 3)$. Keeping a prescribed free mass m gives

$$t(a) = \frac{1}{2(3 + e^{am})} = \frac{1}{8} - \frac{am}{32} + O(a^2 m^2).$$

This repairs that free mass scaling. It does not prove an interacting electron limit or a Maxwell photon. Approaching the edge of a norm estimate is not a proof that the desired critical continuum exists.

A massless photon does not fix its coupling

In rationalised units with a fixed unit phase charge, adopt a Maxwell quadratic action with positive coefficient κ_R . Its transverse inverse response is $\kappa_R k^2 P_T$. Every positive κ_R gives the same massless pole, while the residue changes. The static equation gives

$$-\kappa_R \nabla^2 a_0 = \delta^3(r), |V(r)| = \frac{1}{4\pi\kappa_R r}, \alpha = \frac{1}{4\pi\kappa_R}.$$

Thus finite charged mass and photon masslessness do not select α . A field redefinition can move κ_R into the charge normalisation but cannot remove the physical dimensionless interaction strength when that charge convention is held fixed.

The intrinsic record rate is another independent freedom in a conditional extension. A dephasing term of rate γ can change visibility by $e^{-\gamma t}$ while leaving the coherent Hamiltonian fixed. At a calibrated time τ , the two observables $(1/(4\pi\kappa_R), e^{-\gamma\tau})$ vary independently with κ_R and $\gamma\tau$. One unspecified scalar closure condition does not automatically determine both.

Closure status and next step

There are explicit noncommuting representations and finite positive gauge models. The Wilson branch has a rigorous uniform obstruction to a proposed fixed parameter refinement. The observed gauge group, matter spectrum, chiral content, continuum limits and dimensionless couplings remain open. No Yang Mills mass gap or fine structure constant prediction is claimed.

A constructive next attempt must freeze M1 and M2 before evaluating its target data, then prove a charged sector, its controlled continuum response and the response coefficient in the same construction. R10 asks the corresponding question for gravitational geometry.

Sources

[S01] *PDT Foundations and Physical Closure*, 12 September 2026, Section 15.

[S07] *PDT Physical Closure Gap and Refinement Test*, 18 September 2026. The Wilson conventions, uniform decay proof, free tuning formula and independent response freedoms are retained from this source.

[B10] K. G. Wilson, “Confinement of Quarks”, *Physical Review D* 10, 2445–2459 (1974). DOI. The finite gauge action is standard lattice gauge theory mathematics, used here as a conditional witness.