

Cosmology and effective stress

PDT Foundations R11

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Purpose and dependencies

Cosmological names cannot replace a stress tensor. This paper states the requirements for interpreting a relational component as matter, radiation or accelerated expansion. It also constructs a nonempty classical cosmological completion and proves a future existence result under explicit assumptions. The physical origin of those assumptions remains open.

Use the conditional Einstein dynamics of **R10**, with $c = \hbar = 1$ and $M^2 = (8\pi G)^{-1}$. The premise **K2** supplies homogeneous isotropic geometry and initial data. **K3** supplies the actual matter action, stress and any interactions. A scalar field example below is one K3 choice. Naming that scalar a phase field does not derive its action from A1.

Conservation determines dilution

For a Friedmann geometry $ds^2 = -dt^2 + a(t)^2 d\Sigma_k^2$, use dimensionless spatial coordinates, dimensionless $k \in \{-1, 0, 1\}$ and a scale factor with length units. Let ε denote energy density and p pressure. The homogeneous equations are

$$H^2 = \frac{\varepsilon}{3M^2} + \frac{\Lambda}{3} - \frac{k}{a^2}, \dot{\varepsilon} + 3H(\varepsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{\varepsilon + 3p}{6M^2}, H = \dot{a}/a.$$

These equations assume the chosen common stress is conserved. A separately conserved component with constant $w = p/\varepsilon$ therefore satisfies $\varepsilon \propto a^{-3(1+w)}$, by integrating $d \log \varepsilon = -3(1+w) d \log a$.

In an isotropic rest frame, a traceless stress obeys $T^\mu_\mu = -\varepsilon + 3p = 0$. For nonzero positive density it consequently has $w = 1/3$ and radiation dilution a^{-4} . It cannot simultaneously be pressureless cold matter or a constant vacuum density under those same assumptions.

Proposed component	Required background behaviour	Further physical test
Cold dark matter	Small pressure and approximately a^{-3} dilution	Clustering, sound speed, abundance and lensing
Relativistic transport	$w = 1/3$ if isotropic and traceless	Radiation density and free streaming effects
Accelerating component	Total $\varepsilon + 3p < 0$ when $\Lambda = 0$	Perturbations, stability and expansion history

Interactions can alter separate dilution laws, but then the energy transfer terms must appear explicitly and sum to zero in the total continuity equation. They cannot be used as an unspecified exception after a scaling failure.

A scalar illustration

For a canonical homogeneous scalar with potential $V(\phi)$,

$$\varepsilon_\phi = K + V, p_\phi = K - V, K = 1/2 \dot{\phi}^2, \\ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0.$$

If it dominates, $\Lambda = 0$ and its density is positive, acceleration requires $K < V/2$. Pure kinetic drift has $w = 1$, not negative pressure. A nearly stationary positive potential has w near minus one. Thus residual phase drift is not a dark energy derivation without a specified action and stress.

A rapidly oscillating massive canonical field in a quadratic potential can instead have averaged pressure near zero. Over oscillations short compared with the expansion time, multiplying its equation by ϕ and averaging gives $\langle \dot{\phi}^2 \rangle \simeq m^2 \langle \phi^2 \rangle$, with expansion corrections. This is a conditional mechanism for matterlike behaviour. It requires the mass, potential, initial population and controlled averaging regime. It does not follow from the word transport.

An exact expanding witness

Take $k = 0$, $\Lambda = 0$, conserved dust with $\varepsilon_m = C/a^3$ and a positive constant potential V_0 . Define $H_\Lambda = \sqrt{V_0/(3M^2)}$. Then for $t > 0$,

$$a(t) = \left(\frac{C}{V_0} \right)^{1/3} \sinh^{2/3} \left(\frac{3}{2} H_\Lambda t \right).$$

To verify it, put $u = 3H_\Lambda t/2$. Its Hubble rate is $H_\Lambda \coth u$, while $\varepsilon_m = V_0 \operatorname{csch}^2 u$. The identity $\coth^2 u = 1 + \operatorname{csch}^2 u$ proves the first Friedmann equation. Dust conservation and constant V_0 then give the acceleration equation by differentiation.

This is a consistent supplied matter plus vacuum cosmology. Its dust and potential have not been derived from primitive phase relations. The constant contribution is counted once, through V_0 , while Λ has been set to zero. The singular limit $t \rightarrow 0$ of this classical solution is not a derivation of the origin of the universe or a resolution of the Big Bang.

A conditional future existence theorem

Theorem R11.1. Consider the flat expanding scalar plus dust system with $\Lambda = 0$, nonnegative initial dust density, $a_0 > 0$, and a twice continuously differentiable coercive potential satisfying $V \geq V_{\min} > 0$. For finite initial scalar data satisfying the Friedmann constraint, the solution exists at every finite future time. It remains expanding, dust decays to zero and

$$\int_{t_0}^{\infty} \dot{\phi}^2 dt < \infty.$$

Proof. The scalar energy $E_\phi = \dot{\phi}^2/2 + V$ satisfies $\dot{E}_\phi = -3H\dot{\phi}^2 \leq 0$. The Friedmann constraint gives $H \geq \sqrt{V_{\min}/(3M^2)} > 0$. The scalar sublevel set is compact by coercivity, and the scalar velocity is bounded by its initial energy. Dust decreases through $\dot{\epsilon}_m = -3H\epsilon_m$. Total density is bounded, hence H is bounded above as well. On every finite future interval, a stays positive and finite, and all state variables remain in a region where the ODE vector field is locally Lipschitz and bounded. The ordinary continuation theorem gives global future existence. The positive lower bound on H makes a grow at least exponentially and dust tend to zero. Integrating the scalar energy equation and using the same lower bound proves the finite integral. ■

This theorem establishes a nonempty, controlled homogeneous evolution class. It does not address past completeness, nonlinear inhomogeneous stability, reheating or the observed primordial spectrum. Inflation requires an accelerating phase plus a specified exit, perturbation dynamics and comparison with observations. A positive potential alone does not provide that full account.

Radiation abundance as a sharper check

If a new relational component is radiationlike during nucleosynthesis or recombination, its abundance matters as well as its equation of state. In the conventional late radiation parametrisation,

$$\epsilon_{rad} = \epsilon_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{eff} \right].$$

Planck 2018 combined with baryon acoustic oscillation data reported $N_{eff} = 2.99 \pm 0.17$ in the stated extended cosmological model [B16]. This is a dated CMB plus BAO benchmark, not a model independent bound on every possible early component. A PDT comparison must supply the component's production, temperature or distribution, interactions and perturbation behaviour. A component present only in a different era needs the appropriate analysis.

Status and sources

The dilution and trace constraints, the exact expanding solution and the future existence theorem are conditional results. Physical dark matter, dark energy, inflation, initial conditions and a distinctive PDT cosmological prediction remain unselected. The main book should use the same status wherever it discusses these mechanisms.

[S01] *PDT Foundations and Physical Closure*, 12 September 2026. Geometry and physical identification boundaries.

[S07] *PDT Physical Closure Gap and Refinement Test*, 18 September 2026. Distinction between a specified cosmological evolution and independent selection of its action.

[B16] Planck Collaboration, “Planck 2018 Results VI Cosmological Parameters”, *Astronomy and Astrophysics* 641, A6 (2020). [arXiv 1807.06209](https://arxiv.org/abs/1807.06209).

[B17] S. M. Carroll, *Lecture Notes on General Relativity*. [arXiv gr-qc/9712019](https://arxiv.org/abs/gr-qc/9712019). Curvature and homogeneous Einstein equations in the stated conventions.